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MASS TRANSFER OF A SINGLE BUBBLE IN A MINIMALLY FLUIDIZED GRANULAR BED

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The transfer of a gas admixture through the boundary of a cloud of closed circulation of gas is examined with a view to both molecular and convective dispersion.

Mass transfer of gas bubbles with a dense phase of the fluidized bed largely determines the effectiveness of operation of catalytic chemical reactors and other industrial equipment with granular material. Therefore, in addition to the accumulation of experimental data, simple models of mass transfer were also suggested; some of them were explained in [1-10]. However, the problem of the theoretical determination of the corresponding mass-transfer coefficient and its dependence on the physical and regime parameters is far from being satisfactorily solved; this is undoubtedly due to the variety of phenomena of differing physical nature affecting the process of mass transfer.

The numerous difficulties marking the problem of exchange of a single bubble with a single-phase liquid are in our case compounded by the fact that the dense phase of the fluidized bed is gas-permeable. This leads to the appearance of supplementary convective flows and complicates the purely hydrodynamic part of the problem: before the problem of mass transfer itself is solved, it is necessary to construct an acceptable model of the motion of both phases in the vicinity of the bubble; this in itself is a very nontrivial problem [1, 11, 12]. Two fundamentally different regimes of bubble motion are possible: with a cloud of closed circulation of gas and without it; the nature of the mass transfer will then also be fundamentally different.

Furthermore, in the gas stream permeating the dense phase, the compressibility of the process of molecular diffusion of the admixture is important, i.e., a magnitude of the type of the known coefficient of sinuousness has to be introduced. moreover, additional convective dispersion appears due to the mixing of elementary jets in the intersected pore space of a moving porous body formed by the moving particles which macroscopically can be described as a diffusion-type random process (see, e.g., [13, 14]). This dispersion is already substantial for beds with particles of $\sim 10^{-2}$-cm diameter; as a result, the effective diffusion coefficient in the vicinity of the bubble is nonuniform, being dependent on the particle size and the local porosity of the dense phase and on the relative gas velocity.

The supplementary transfer of the gaseous admixture in the general case is effected by particles that absorb or adsorb it, and in this process particles participate that belong to the dense phase as well as those that come through the bubble [5, 6, 15]. Moreover, adsorption of the admixture by the particles, as well as chemical reactions with its participation, obviously affect the convective diffusion of the admixture in the gas phase.

Finally, serious difficulties are also posed by the necessity of expressing the nonsteadiness of the process of mass transfer. In addition to non-steady-state effects connected with the establishment of the steady-state regime and ceasing to be substantial after a certain time interval since the beginning of the process has passed, exceeding the value $2R/U$, where R is the order of magnitude of the bubble radius or of the cloud of closed circulation around it, there appear effects that have no analog in the mass transfer of a bubble in a single-phase liquid. Firstly, a real bubble in a fluidized bed changes its volume in accordance with its lift, and this gives rise to a radial gas stream affecting the mass transfer [9, 10]. Secondly, the random pulsations of the bubble obviously lead not only to some nonsteadiness of the hydrodynamic fields but also to the "detachment" of parts of the cloud together with the gas contained in them; this is bound to intensify the mass transfer [4].

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Taking into account the collective effects in mass transfer of a large number of bubbles in a fluidized bed is also more complicated than the analogous problem for bubbles in a single-phase liquid, mostly because of the more favorable conditions for the coalescence of bubbles affecting the mass transfer [5, 16].

At present it is impossible to describe all these effects within the framework of one model, and it is apparently expedient to examine them separately. The present work examines the steady-state problem of the mass transfer of a single bubble with a gas cloud of closed circulation in a minimally fluidized bed (where a change in the volume of the bubble may be disregarded [10]), neglecting the random "detachment" of parts of the cloud and the absorption of the admixture by particles or its transformation as a result of chemical reactions; attention centers on describing the transfer of the admixture through the outer boundary of the cloud by the mean motion of gas, as well as both molecular and convective dispersion.

Simple estimates show that it is practically always possible to assume that the Peclet number, calculated by the radius and lift velocity of a bubble and the effective diffusion coefficient in the solid phase ($D \sim 0.1$-1 cm^2/sec, $R_B \sim 1$-10 cm), is considerably larger than unity.* Under these conditions the concentration of the admixture outside the system cloud–bubble has to be uniform everywhere with the exception of a thin boundary "diffusion" layer adjacent to the boundary of the system. At the initial stage of the process of mass transfer, an analogous statement is also correct for the region bordering on the inside on the boundary of the cloud: the concentration of the admixture in the cloud is nonuniform within a thin "inner" diffusion layer [17, 18]. At this stage, both diffusion layers are equally important, as was noted in [10], and they have to be taken into account in the analysis. However, when this period is concluded, the circulatory nature of the gas flow along the closed flow lines inside the bubble and the cloud becomes important, when "the basic condition of the existence of a boundary layer (constant concentration at the core of the flow) is not fulfilled." This was discussed in [19] for the internal problem of mass transfer involving a drop.

Obviously, the duration of the initial stage is proportional to 2R/U, and the characteristics of mass transfer obtained from the solution of the corresponding problem with two diffusion layers represent the lower boundaries of the true characteristics. Their upper boundaries, attained at instants far byond the time of circulation, may be obtained on the assumption that there is complete mixing inside the system cloud–bubble, when there is only an outer diffusion layer. In both cases, with fairly large bubbles and sufficiently small particles, when there is a region of closed gas circulation, the chief resistance to mass transfer is concentrated on the outer boundary of the cloud. Below we first solve the problem with two diffusion layers.

As the initial hydrodynamic model of flow around a bubble we use the model from [1] in which the property outside the bubble is taken as uniform, and in the system of spherical coordinates, associated with the center of the bubble, we have

$$\psi = (U - u_0)\left(1 - \frac{R_c^3}{r^3}\right)\frac{r^2}{2}\sin^2\theta, \quad \left(\frac{R_c}{R_B}\right)^3 = \frac{U + 2u_0}{U - u_0} \tag{1}$$

($\theta = \pi$ corresponds to the frontal point). The flow function (1) determines the mean velocity **v** of the gas in the gaps between particles whose components are equal to

$$v_r = (U - u_0)\left(1 - \frac{R_c^3}{r^3}\right)\cos\theta, \; v_\theta = -(U - u_0)\left(1 + \frac{R_c^3}{2r^3}\right)\sin\theta. \tag{2}$$

The components of the mean particle velocity are represented in the form

$$w_r = U\left(1 - \frac{R_c^3}{r^3}\right)\cos\theta, \; w_\theta = -U\left(1 + \frac{R_c^3}{2r^3}\right)\sin\theta. \tag{3}$$

Furthermore, we also need the components of the relative gas velocity $\mathbf{u} = \mathbf{v} - \mathbf{w}$ for $r = R_c$, which, according to (2) and (3), are equal to

$$u_r = \frac{3u_0U}{U + 2u_0}\cos\theta, \; u_\theta = \frac{3u_0^2}{U + 2u_0}\sin\theta. \tag{4}$$

*The conclusion in [7] that the Peclet numbers for real fluidized systems are smaller than unity is inaccurate because in the estimates effective diffusion coefficients of the order of magnitude of 100 cm^2/sec were used. This conclusion was apparently reached because the process of small-scale dispersion (with an effective coefficient of the order of magnitude of 0.1-1 cm^2/sec) was identified with the large-scale mixing of the gas in the apparatus as a whole, connected not so much with small-scale dispersion but rather with mixing caused by the passage of the bubbles and by circulation currents.

The magnitudes u_0 and U may be written in the form*

$$u_0 = \frac{u_*}{\varepsilon}, \quad U \approx 0.9(1-f)^{1/6}(gR_B)^{1/2}. \tag{5}$$

We will now examine the small-scale dispersion of the admixture inside the dense phase. According to [22], the flow of the admixture that is not absorbed by the particles can be represented in the form

$$\mathbf{q} = -D_m \nabla c + \langle \eta c' \mathbf{v}' \rangle, \tag{6}$$

where the structural function η is equal to unity in the gaps between the particles and to zero inside them, and the prime denotes pulsation of the magnitudes around their mean values. In the general case, the effective coefficient D_m of molecular diffusion in the gas phase of the dispersed medium may depend not only on the shape and orientation of the particles D_0 and ϵ, respectively, but also on the parameters characterizing the flow around individual particles of the dispersed phase and the mean motion of the medium [23].

The second term on the right-hand side of (6) describes the convective dispersion of the admixture caused by the mixing of the elementary jets appearing upon flow around the regularly situated particles as well as by random "pseudo-turbulent" motions connected with the fluctuations of porosity in the system. In a motionless granular bed these fluctuations originate as a result of random deviations of the real packing from the idealized regular packing, in the fluidized bed on account of the chaotic motion of the suspended particles; such fluctuations were investigated theoretically in [24].

With the aid of a hypothesis of the type of the well-known Prandtl hypothesis about the length of mixing in the theory of turbulence, the mentioned term is transformed to the form $\mathbf{D}_c \nabla c$, where $\mathbf{D}_c$ is the tensor magnitude, one of whose principal axes has the direction of the local mean relative gas velocity $\mathbf{u}$, and whose other two axes are arbitrarily orientated in a plane normal to $\mathbf{u}$ (see [13, 14], and also the review of empirical data in [25]). The corresponding eigenvalues of this tensor determine the coefficients of longitudinal and transverse convective dispersion, which are usually written in the form

$$D_{c,\parallel} = k_\parallel 2a\varepsilon u, \quad D_{c,\perp} = k_\perp 2a\varepsilon u, \tag{7}$$

where in the general case, $k_\parallel$ and $k_\perp$ may also depend on the Reynolds and Peclet numbers for one particle [14, 25]. For idealized packing of particles, the model in [13] yields for these coefficients the constant values $k_\parallel = 0.76$, $k_\perp = 0.18$, which do not differ very much from the empirical values [14, 25].

With a view to the approximate nature of the developed theory, due mainly to the approximate nature of the hydrodynamic model in [1], which leads to the formulas (1)-(4), we neglect the dependence on the local hydrodynamic situation at the level of individual particles of the values $k_\parallel$ and $k_\perp$ (which corresponds to neglecting the pseudoturbulent convective dispersion) and of the coefficient D_m (which corresponds to using the known hypothesis of superposition). Then, if we estimate the coefficients in (7) on the basis of [13], and use the theory of [26], well confirmed by experiments, for calculating D_m, we obtain from (6) and (7) that

$$\mathbf{q} = -\mathbf{D}\nabla c, \quad D_\parallel = \beta D_0 + 1.52a\varepsilon u, \quad D_\perp = \beta D_0 + 0.36a\varepsilon u,$$
$$\beta = (17+7\rho)^{-1}\{5 - 11\rho + [(5-11\rho)^2 + 7(1-\rho)(17+7\rho)]^{1/2}\}, \quad \rho = 1-\varepsilon. \tag{8}$$

In particular, the radial flow of admixture at the boundary of the cloud (i.e., with $r = R_c$) on the basis of (8) is written as follows:

$$q_r = -(\beta D_0 + 1.52a\varepsilon|u_r| + 0.36a\varepsilon|u_\theta|)\left(\frac{\partial c}{\partial r}\right), \tag{9}$$

*In the minimally fluidized state, u_* coincides with the velocity of fluidization. The presented formula for the lift velocity of the bubble follows from the experimentally established correlation $U \approx 0.71 g^{1/2} V^{1/6}$ in [1], where V is the true volume of the bubble on the assumption that the part of the volume of the sphere with radius R_B, coinciding with the radius of the frontal part of the bubble, taken up by the wake is equal to f. The frequently used relationship $U \approx 0.71(2gR_B')^1 \approx (gR_B')^{1/2}$, where $R_B' < R_B$ is the radius of the sphere with equal volume as the bubble, follows from the Davis–Taylor formula $U = (2/3)(gR_B)^{1/2}$, if f $\approx$ 0.915, as was found for large bubbles in a single-phase liquid. In a fluidized bed f is many times smaller (according to data of [20], e.g., f $\approx$ 0.25–0.35, Chiba and Kobayashi [5] took f = 0.25). We therefore gave preference here to Eq. (5). However, the difference between these formulas is not large (this is due to the fact that the value 1 − f figures with a small power in them, 1/6) and it is possibly within the experimental error. We also point out that the formula in (5) applies to steady-state motion of the bubble and is correct for time of the order of magnitude 2R/U after the beginning of the motion [21].

where u_r and u_θ are determined in (4).*

The effective coefficient of radial diffusion, figuring in (9), is nonuniform on the surface of the cloud. Taking relations (4) into account, we see that the second term of the expression for this coefficient usually predominates over the third term, i.e., the coefficient has maxima when the values of θ are very close to 0 and π, and minima when $\theta = \pi/2$.

The equations of steady-state convective diffusion on both sides of the outer boundary of the cloud are written in the form

$$\varepsilon\left(v_r\frac{\partial}{\partial r}+\frac{v_\theta}{r}\frac{\partial}{\partial\theta}\right)\begin{Bmatrix}c\\c'\end{Bmatrix}=\nabla\left(\begin{Bmatrix}D\\D'\end{Bmatrix}\nabla\begin{Bmatrix}c\\c'\end{Bmatrix}\right), \tag{10}$$

where the symbols with primes refer to the region lying inside the outer boundary of the cloud; with $r = R_c$, the conditions of continuity of concentration and of the normal component of the diffusion flux have to be satisfied.

In principle, two extreme situations are possible. In one of them the characteristic thickness of the inner diffusion layer (let us say, at the frontal point of the bubble) is much smaller than the thickness of the cloud, so that this layer lies practically entirely (with the exception of a small region near the wake that plays no particular part in the mass transfer) inside the cloud. In that case, D' is determined with the aid of the same functional relation as **D**. In the second situation the cloud is considerably thinner than the diffusion layer, and D' is a spherical tensor with eigenvalue equal to D_0. We will examine only the first situation because it is much more realistic for bubbles characterized by large Peclet numbers.

Using the approximation of a thin diffusion boundary layer [17], and also expressions (1), (2), (4), and (9), we transform (10) with the aid of standard methods into the classical diffusion equation

$$\frac{\partial}{\partial\xi}\begin{Bmatrix}c\\c'\end{Bmatrix}=\frac{\partial^2}{\partial\psi^2}\begin{Bmatrix}c\\c'\end{Bmatrix}. \tag{11}$$

Here we use the new independent variables ψ from (1) and

$$\begin{aligned}\xi &= \frac{3}{2}(U-u_0)DR_c^3\int_\theta^\pi(1+\gamma|\cos\theta|+\gamma'\sin\theta)\sin^3\theta d\theta = \\ &= \frac{1}{2}(U-u_0)DR_c^3\left\{2+3\cos\theta-\cos^3\theta+\frac{3}{4}[\gamma p(\theta)+\gamma' s(\theta)]\right\},\end{aligned} \tag{12}$$

and we introduce the functions

$$\begin{aligned}p(\theta) &= \begin{cases}\sin^4\theta, & \pi/2<\theta<\pi,\\ 2-\sin^4\theta, & 0<\theta<\pi/2,\end{cases}\\ s(\theta) &= \frac{3}{2}(\pi-\theta)+\sin 2\theta-\frac{1}{8}\sin 4\theta\end{aligned} \tag{13}$$

and the parameters

$$D=\frac{\beta}{\varepsilon}D_0,\quad \gamma=4.56\frac{\varepsilon a}{\beta D_0}\,\frac{Uu_0}{U+2u_0},\quad \gamma'=1.08\frac{\varepsilon a}{\beta D_0}\,\frac{u_0^2}{U+2u_0}. \tag{14}$$

Equation (11) has to be solved with the ordinary initial and boundary conditions

$$\begin{aligned}&c\to c_0,\ \psi\to\infty;\ c'\to c_0',\ \psi\to-\infty;\ c=c',\ \frac{\partial c}{\partial\psi}=\frac{\partial c'}{\partial\psi},\ \psi=0;\\ &\begin{Bmatrix}c\\c'\end{Bmatrix}=\begin{Bmatrix}c_0\\c_0'\end{Bmatrix},\ \xi=0.\end{aligned} \tag{15}$$

*We point out that in works dealing with the mass transfer of bubbles with a dense phase of the fluidized bed, the presence of a convective component in the effective diffusion coefficient of the gaseous admixture is usually not emphasized although in a bed of fairly large particles it is very important. An exception is the article [27] in which the effective coefficient of radial diffusion in the dense phase, which has to figure in the relation (9), is taken equal to the effective coefficient of transverse diffusion and in the bed as a whole. This is incorrect for two reasons. Firstly, the identity of the small-scale mixing in the dense phase and the large-scale mixing in the entire bed is admitted. Secondly, at the boundary of the cloud, the "longitudinal" (radial) component of the mean relative gas velocity has the same order of magnitude as the "transverse" (tangential) component, and the more intensive longitudinal convective dispersion will play the most important part.

The solution of the problem (11), (15) has the form

$$c = c_0 + \frac{c_0' - c_0}{2} \operatorname{erfc} \frac{\psi}{2\sqrt{\xi}}, \quad c' = c_0' + \frac{c_0 - c_0'}{2} \operatorname{erfc} \frac{-\psi}{2\sqrt{\xi}}. \tag{16}$$

The effective radial flow of the admixture through the outer boundary of the cloud is calculated from (9) and (16), taking into account the determination of ψ and ξ in (1) and (12). As a result we have

$$\begin{gathered} q_r = \frac{3}{2\sqrt{2\pi}} |c_0 - c_0'| \varepsilon \left[\frac{(U-u_0)D}{R_c}\right]^{1/2} f(\theta, \gamma, \gamma'), \\ f(\theta, \gamma, \gamma') = \frac{(1+\gamma|\cos\theta| + \gamma' \sin\theta)\sin^2\theta}{\{2 + 3\cos\theta - \cos^3\theta + (3/4)[\gamma p(\theta) + \gamma' s(\theta)]\}^{1/2}}. \end{gathered} \tag{17}$$

The full flow of the admixture is obtained from this after integration over the sphere $r = R_c$, and it is equal to

$$\begin{gathered} Q = 3\sqrt{\frac{\pi}{2}} |c_0 - c_0'| \varepsilon [(U-u_0)D]^{1/2} R_c^{3/2} F(\gamma, \gamma'), \\ F(\gamma, \gamma') = \int_0^\pi f(\theta, \gamma, \gamma') \sin\theta d\theta. \end{gathered} \tag{18}$$

Also interesting is the distribution of the flow over the surface of the outer boundary of the cloud characterized by the magnitude

$$\frac{q_r}{\langle q_r \rangle} = \frac{4\pi R_c^2 q_r}{Q} = \frac{2f(\theta, \gamma, \gamma')}{F(\gamma, \gamma')}, \tag{19}$$

where f and F are determined in (17) and (18), respectively.

The condition of correctness of the assumption that the inner diffusion layer with practically all values of θ lies inside the cloud, i.e., in a region occupied by the dispersed medium, assumes the following form:

$$\frac{1}{\mathrm{Pe}} = \frac{D}{UR_c} \ll \frac{U-u_0}{U}\left[1 - \left(\frac{U-u_0}{U+2u_0}\right)^{1/3}\right]^2. \tag{20}$$

For large bubbles ($U \gg u_0$) we obtain

$$\frac{1}{\mathrm{Pe}} = \frac{D}{UR_c} \ll \left(\frac{u_0}{U}\right)^2, \tag{21}$$

and it can be seen that with sufficiently large Peclet numbers, when the approximation of the thin diffusion layer is justified, this assumption is fulfilled for bubbles of all sizes, with the exception of the very smallest for which U is comparable with u_0.

The dependence of F from (18) on γ, with different γ', is shown in Fig. 1, from which it can be seen that the existence of convective dispersion substantially intensifies the mass transfer. Figure 2 presents the dependences of the values from (19) on θ for different γ and $\gamma' = 0$. It can be seen that the presence of a minimum of the coefficient of radial diffusion in (9) causes a relative weakening of mass transfer in the equatorial zone of the bubble $\theta \sim \pi/2$. Figure 3 shows the effect of the contribution from the transverse convective dispersion on the distribution of the local flow over the outer surface of the cloud: as was to be expected, it leads to a certain smoothing of the previously mentioned minimum of the local flow in the equatorial zone.*

*As was pointed out in [28], the nonuniformity of the effective diffusion coefficient leads to the appearance of new effects. Yet in the examined problem of mass transfer of a bubble they are in some sense inverse to those of [28], where the problem of the heat and mass transfer of a solid body submerged into an infiltrated granular bed was investigated. As distinct from the situation examined here, where the diffusion coefficient is minimal near the equator of the bubble in the flow, in [28] this coefficient attained its maximum at the equator of the sphere in the flow.

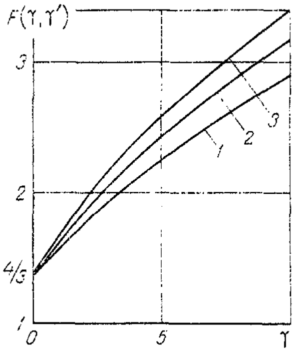

Fig. 1

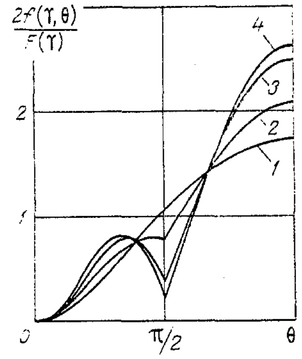

Fig. 2

Fig. 1. Dependence of the coefficient F, determining the intensity of mass transfer between the bubble and the dense phase, on the parameters γ and γ' characterizing the specific weight of the longitudinal and transverse convective dispersions, respectively: 1) $\gamma' = 0$; 2) $\gamma' = 0.1\gamma$; 3) $\gamma' = 0.2\gamma$.

Fig. 2. Distribution of local flow over the surface of the closed-circulation cloud with $\gamma' = 0$ and different γ: 1) $\gamma = 0$; 2) 1; 3) 5; 4) 10.

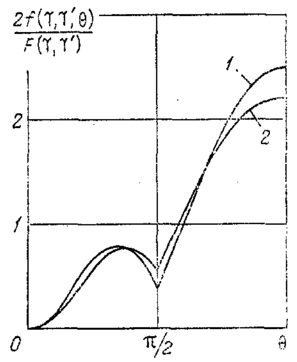

Fig. 3

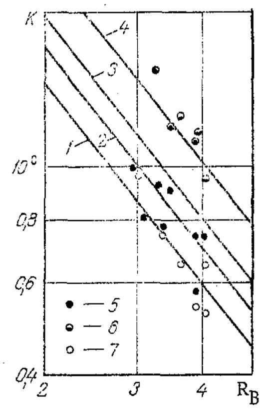

Fig. 4

Fig. 3. Effect of the transverse convective dispersion on the distribution of the local flow: $\gamma = 5$, $\gamma' = 0$ (curve 1), and $\gamma' = 0.2\gamma$ (curve 2).

Fig. 4. Dependence of the mass-transfer coefficient K on the bubble radius with $u_0/U = 0$, $\gamma' = 0$, $\epsilon = 0.5$, and different γ: 1) $\gamma = 0$; 2) 1; 3) 2; 4) 5; the dots correspond to the experiments in [5]: 5) glass beads; 6, 7) crushed glass with $a = 1.05 \cdot 10^{-2}$ cm and $0.7 \cdot 10^{-2}$ cm, respectively; $D_0 = 0.2$ cm^2/sec.

The relations obtained above apply to the case when both diffusion boundary layers are substantial, and they describe the lower boundaries for the true local and full flows q_r and Q, respectively. The upper boundaries for these flows, corresponding to the regime with one diffusion layer established fairly high above the gas-separating screen, are twice as high as the lower ones. They can also be characterized by the curves in Figs. 1-3.

The obtained results make it possible to express in explicit form various mass-transfer coefficients that were previously introduced into the literature and were distinguished by the fact that they correlate the flow of admixture Q with unit volume or unit surface of a real bubble, of a sphere with radius R_B or a sphere with radius R_c, or else (if we deal with a number of bubbles) with unit specific surface (per bubble) of the layer as a whole or of its dense phase. For instance, the mass-transfer coefficient introduced in [5] into the calculation per unit volume of the bubble is equal to (here and below we have in mind the upper boundary for characteristics of mass transfer)

$$K = \frac{Q}{(4/3)\pi R_B^3 (1 - f)|c_0 - c_0'|} = L\left(\frac{UD_0}{R_B^3}\right)^{1/2},$$

$$L = \frac{9F\varepsilon}{2\sqrt{2\pi}(1-f)}\left(\frac{\beta}{\varepsilon}\right)^{1/2}\left(1+\frac{2u_0}{U}\right)^{1/2}. \qquad (22)$$

This coefficient decreases with increasing size of the bubble (in the limit case, as $R_B^{-5/4}$, when $U_0/U = 0$) and increases with increasing particle size, which is accompanied by an increase of u_0 and γ (or F).

To compare the developed theory with experiments, it is necessary to have data from experiments with single bubbles in a bed that is in a state close to minimum fluidization, where it would be guaranteed that there is no absorption of the gaseous admixture by particles and no chemical reactions. From among the experiments in [1-5, 15, 16, 27], and also in [29, 30], this condition is fulfilled only by the experiments in [5], which were well correlated by a formula of the type of (22) with $L \approx 2.39\epsilon(1-f)^{-1}$ for $f \approx 0.25$ and $D = D_0 = 0.205$ cm²/sec. For the beds of glass beads ($a = 1.05\cdot10^{-2}$ cm, $u_* = 3.9$ cm/sec, $\epsilon = 0.49$) and of particles of crushed glass ($a = 1.05\cdot10^{-2}$ cm, $u_* = 5.0$ cm/sec, $\epsilon = 0.53$, and $a = 0.70\cdot10^{-2}$ cm, $u_* = 3.1$ cm/sec, $\epsilon = 0.54$) investigated in [5] we have, according to [5], L = 1.56, 1.70, and 1.72, respectively. In reality, the coefficient L from (22) is a slowly changing function of the particle size. For $R_B = 3$ cm, the corresponding values of the parameter γ from (14) are equal to 1.6, 1.9, and 0.9, respectively, and for L the calculation yields 1.8, 2.0, and 1.9, respectively. Within the experimental error in [5], these values are satisfactory, as are those following from the formula in [5].* Calculation according to the theory of work [7], with $D = D_0$ and $f \approx 0.25$, yields the exaggerated result $L \geqslant 3.2$ (the equality sign corresponds to $u_0/U = 0$).

To illustrate the obtained relations better, Fig. 4 shows the curves $K = K(R_B)$, corresponding to $(u_0/U) = 0$, $\gamma' = 0$, and different values of γ. The figure also contains the experimental data of article [5].

NOTATION

a, particle radius; c, concentration of the admixture; **D**, $\mathbf{D}_c$, tensors of effective diffusion coefficients and of coefficients of convective dispersion, respectively; D, parameter in (14); D_0, D_m, coefficients of molecular diffusion, respectively not taking and taking compressibility into account; F, f, functions in (17) and (18), respectively; f, fraction of the volume of the sphere with radius R_B occupied by the wake of the bubble; g, acceleration of gravity; K, mass-transfer coefficient determined in (22); k, coefficient in (7); L, coefficient in (22); p, s, functions determined in (13); Q, q, flow of admixture for the entire bubble and local flow, respectively; R_B, R_c, radii of bubble and of cloud of closed circulation, respectively; r, radial coordinate; U, lift velocity of the bubble; $\mathbf{u} = \mathbf{v} - \mathbf{w}$; $u_0 = u_*/\varepsilon$; u_*, minimum velocity of fluidization; V, volume of bubble; **v**, **w**, mean gas velocities in the gaps between particles and of the particles, respectively; β, parameter determined in (8); γ, γ', parameters from (14); ϵ, porosity of the dense phase and of the cloud; θ, polar angle; ξ, independent variable introduced in (12); $\rho = 1 - \epsilon$; ψ, flow function.

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*As regards the formula in [5] we want to point out that it is inaccurate for two reasons. Firstly, in its derivation the radial component of the convective term on the left-hand side of Eq. (10) was neglected; this is somewhat reminiscent of the error which was made in theoretical works on the hydrodynamic boundary layer before Prandtl's theory appeared. Secondly, in [5], like in many other works, the compressibility of molecular diffusion as well as the existence of convective dispersion and the corresponding contribution to the effective diffusion coefficient were neglected.

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